

Highlights of this textbook

- ★ **Introduction to graph theory:** The main ideas of graph theory and graph search are explained in Chapter 2, algebraic graph theory in Chapter 5, and bipartite and random graphs in Chapter 10.
- ★ **Heuristic search in artificial intelligence:** Since discrete decision problems have generally large search spaces, the A* search method is introduced in Chapter 3, which prefers promising search directions.
- ★ **Graph-theoretical structure of uncertain knowledge:** Bayesian networks represent the conditional stochastic independence of propositions. Reasoning along their edges utilises the structure of the probabilistic information (Chapter 4).
- ★ **Decomposition and aggregation of interconnected systems:** The coupling graph of interconnected systems introduced in Chapter 6 reveals which systems are decomposable into a series-parallel configuration, which simplifies analysis and design steps.
- ★ **Input-output behaviour of interconnected systems:** For signal-flow graphs, an extension of Cramer's rule for the solution of linear equations leads to Mason's formula to get the transfer function between selected input and output signals (Chapter 6).
- ★ **Generic properties of linear systems:** Controllability and observability are generic properties of dynamical systems, which mainly depend upon the structure graph investigated in Chapter 7.
- ★ **Graph-theoretical modelling and analysis of electrical networks:** Graph theory provides the basic steps of a branch current analysis or a node voltage analysis for modelling electrical networks and for the Kron reduction towards the effective resistance between two terminal nodes (Chapter 8).
- ★ **Maximum flows through networks:** For transportation systems, the max-flow min-cut theorem shows which edges represent the bottleneck of the network (Chapter 8).
- ★ **Structural analysis of discrete-event systems:** The automaton graph introduced in Chapter 9 is the basis of powerful tools for the analysis of nondeterministic and stochastic automata, Markov chains and hidden Markov models. In particular, the Viterbi algorithm finds the most probable path through this graph for a given output sequence.

- ★ **Structural analysis of constraint sets:** Bipartite graphs lead to a decomposition of sets of linear equations to select those variables that are uniquely determined by the model and to find a sequential way for computing these unknown variables (Chapter 11).
- ★ **Structural analysis of fault diagnosability:** For fault diagnosis the bipartite structure graph of a system has to be over-determined to reveal the redundancy that makes a system fault detectable (Chapter 11).
- ★ **Structure design of multi-agent systems:** With graph-theoretical methods the structure of leader-follower systems can be tailored to efficient set-point following and the communication structure can be adapted to the movement of mobile agents as shown in Chapter 12.

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